Streaming Set Cover in Practice

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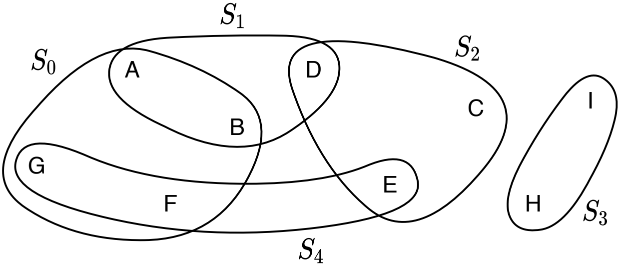
(ALENEX21), January 2021

Example Set Cover Instance



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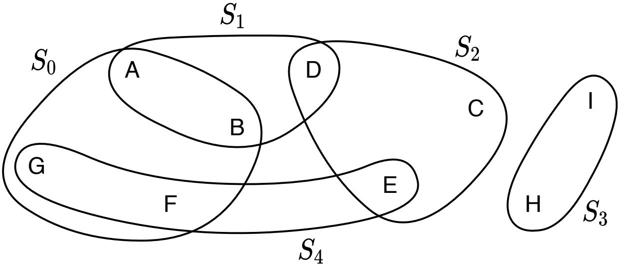

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$\mathcal{S}$	$\{S_0, S_1, S_2, S_3, S_4\}$

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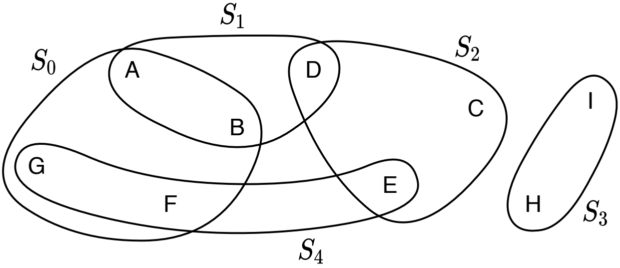

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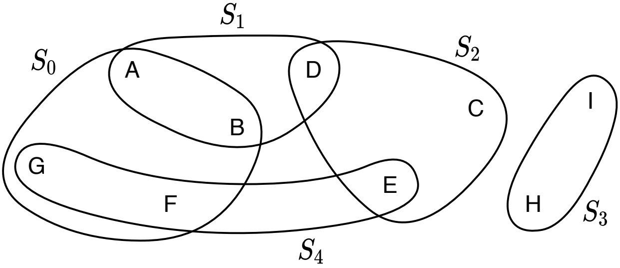

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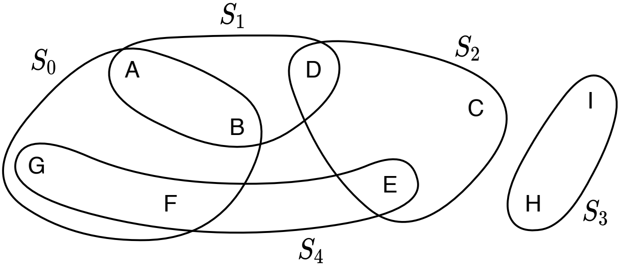

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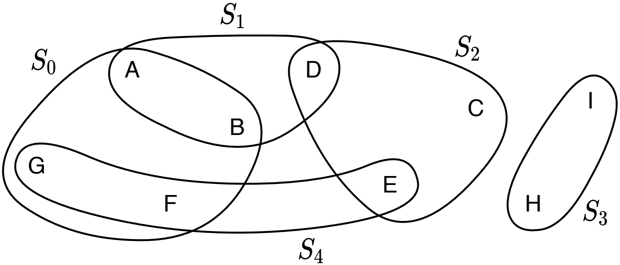

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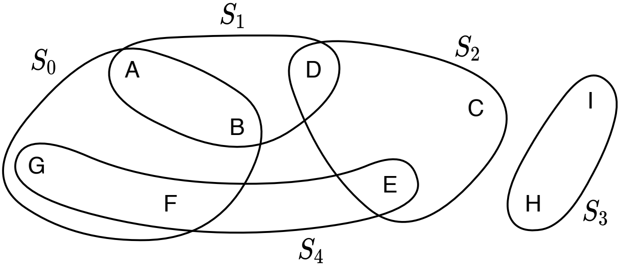

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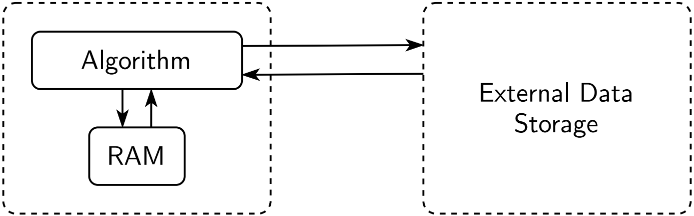
Optimal cover: $\{S_0, S_2, S_3\}$

Memory Access

Direct access:



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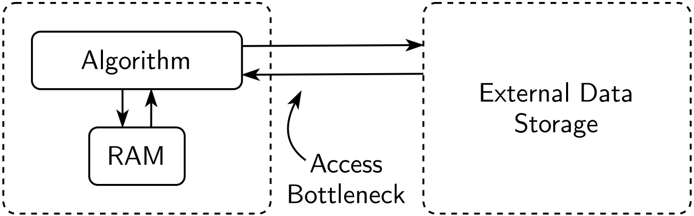
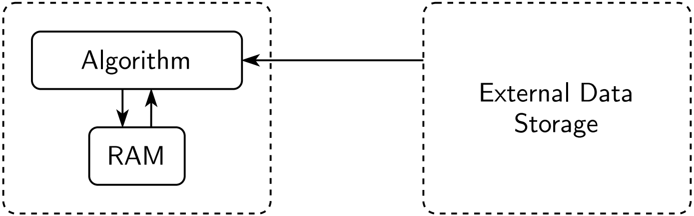
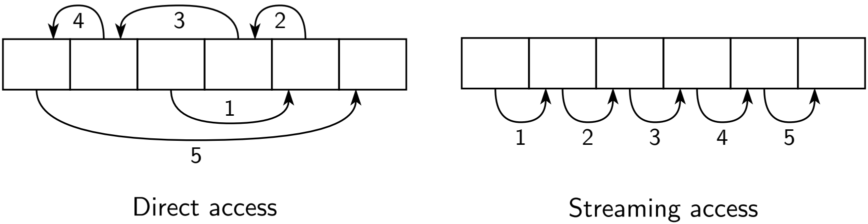


Memory Access

Streaming access:



Accessing Sets



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Approximation factor of $\mathcal{O}(\ln n)$, which is essentially optimal [5, 3]

GREEDY: Implementation

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Alternatives?

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Aim: to access memory in large, contiguous chunks

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- Approximation factor of $1 + p \ln n$ (close to optimal)
- Precomputation step
- Footprint still linear

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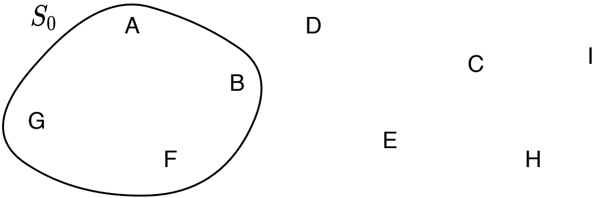

x	A	B	C	D	E	F	G	H	I
eff(x)	-1	-1	-1	-1	-1	-1	-1	-1	-1
eid(x)	-	-	-	-	-	-	-	-	-

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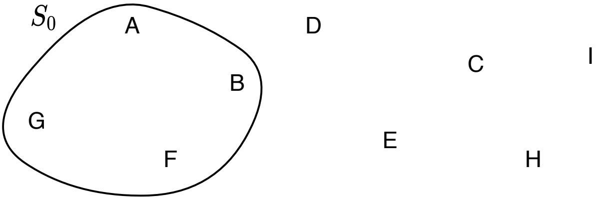

x	A	B	C	D	E	F	G	H	I
eff(x)	**2**	**2**	-1	-1	-1	**2**	**2**	-1	-1
eid(x)	**0**	**0**	-	-	-	**0**	**0**	-	-

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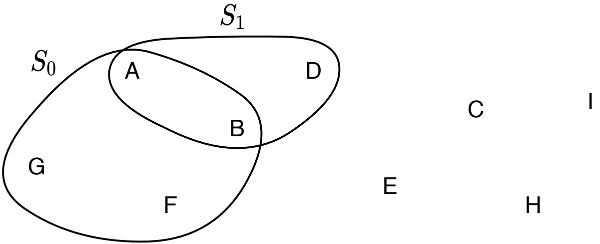

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eff(x)	2	2	-1	-1	-1	2	2	-1	-1
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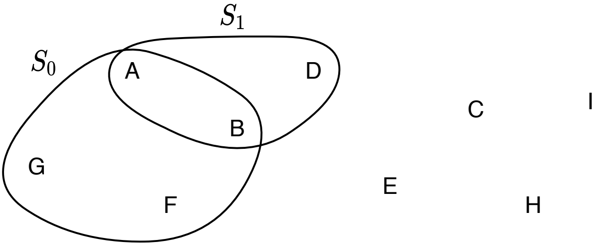

x	A	B	C	D	E	F	G	H	I
eff(x)	2	2	-1	**0**	-1	2	2	-1	-1
eid(x)	0	0	-	**1**	-	0	0	-	-

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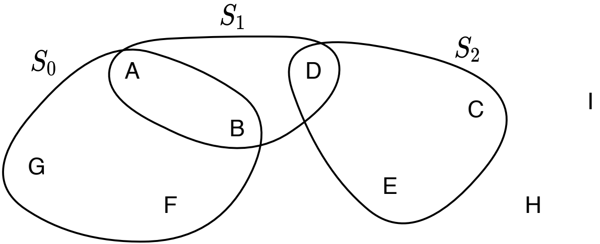

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eff(x)	2	2	-1	0	-1	2	2	-1	-1
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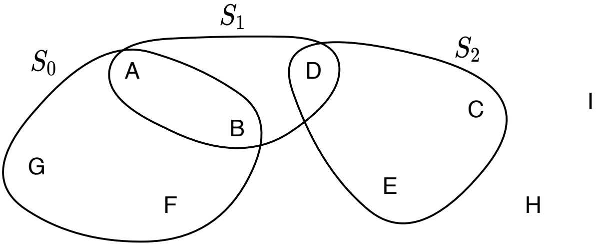

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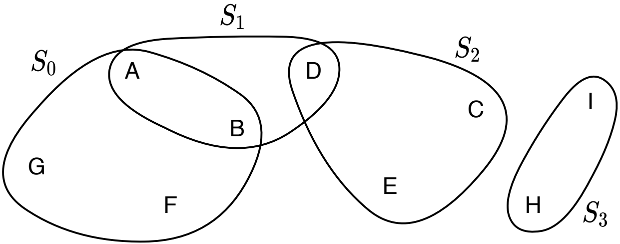

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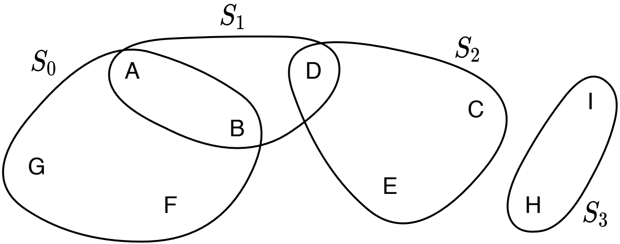

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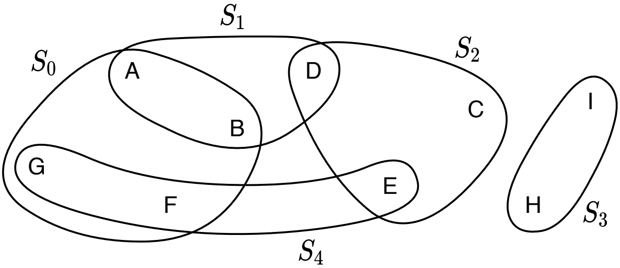

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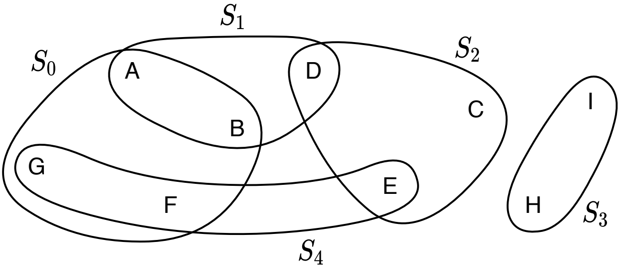

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Aim: use $\widetilde{\mathcal{O}}(n) := \mathcal{O}(n \operatorname{polylog}(n, m))$ bits of working memory

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1. Read set from stream
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- Runs in $\mathcal{O}(M)$ time

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- Runs in $\mathcal{O}(M)$ time
- Uses only $\widetilde{\mathcal{O}}(n)$ space

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How does Emek-Rosén perform in practice?

Emek-Rosén in Practice

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File name	DFG (RAM)	Emek-Rosén
accidents.dat	181	213
kosarak.dat	17 741	18 618
orkut-cmty.dat	149 244	158 439
webdocs.dat	406 338	413 819
twitter.dat	9 246 029	9 955 112
friendster.dat	-	13 310 036

Table: Cover size

Emek-Rosén in Practice

File name	DFG (RAM)	Emek-Rosén
accidents.dat	1.43	0.72
kosarak.dat	1.03	0.79
orkut-cmty.dat	15.44	12.35
webdocs.dat	18.39	15.51
twitter.dat	213.48	158.35
friendster.dat	-	367.52

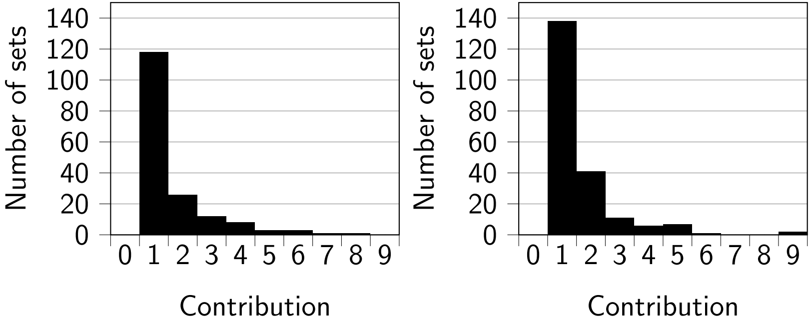
Table: Time (s)

Emek-Rosén in Practice

File name	DFG (RAM)	Emek-Rosén
accidents.dat	66.65	0.91
kosarak.dat	75.45	2.37
orkut-cmty.dat	1094.59	21.67
webdocs.dat	1401.55	56.04
twitter.dat	8044.29	797.70
friendster.dat	-	1183.56

Table: Peak RAM usage (MB)

Emek-Rosén in Practice



(a) DFG (b) Emek-Rosén

Generalising Emek-Rosén to Multiple Passes

Multi-Pass Emek-Rosén

Generalising Emek-Rosén to Multiple Passes

Multi-Pass Emek-Rosén

1. Do an Emek-Rosén pass

Generalising Emek-Rosén to Multiple Passes

Multi-Pass Emek-Rosén

1. Do an Emek-Rosén pass
2. Restrict the universe based on effectiveness

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Multi-Pass Emek-Rosén

1. Do an Emek-Rosén pass
2. Restrict the universe based on effectiveness
3. Repeat for p passes

Generalising Emek-Rosén to Multiple Passes

Multi-Pass Emek-Rosén

1. Do an Emek-Rosén pass
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3. Repeat for p passes

Generalising EMEK-ROSÉN to Multiple Passes

MULTI-PASS EMEK-ROSÉN

1. Do an EMEK-ROSÉN pass
2. Restrict the universe based on effectiveness
3. Repeat for p passes

Approximation factor of $\mathcal{O}(\Delta^{\frac{1}{p+1}})$

Multi-Pass Emek-Rosén in Practice

Multi-Pass Emek-Rosén in Practice

Progressive Greedy [1]

Multi-Pass Emek-Rosén in Practice

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1. Add all sets whose contribution is above a threshold

Multi-Pass Emek-Rosén in Practice

Progressive Greedy [1]

1. Add all sets whose contribution is above a threshold
2. Reduce threshold

Multi-Pass Emek-Rosén in Practice

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1. Add all sets whose contribution is above a threshold
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Multi-Pass Emek-Rosén in Practice

Progressive Greedy [1]

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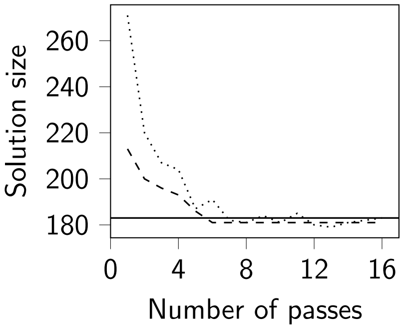
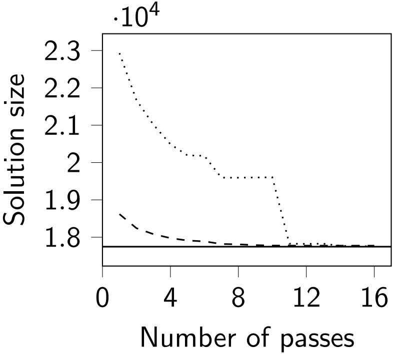
Approximation factor of $\mathcal{O}(\Delta^{\frac{1}{p+1}})$

Multi-Pass Emek-Rosén in Practice

Results (1/3)



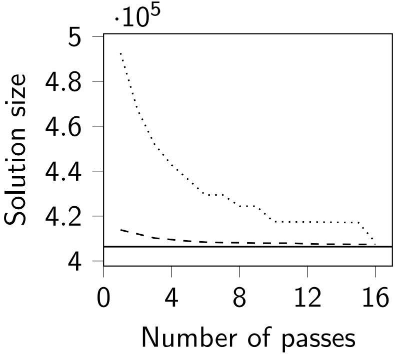
(a) accidents.dat



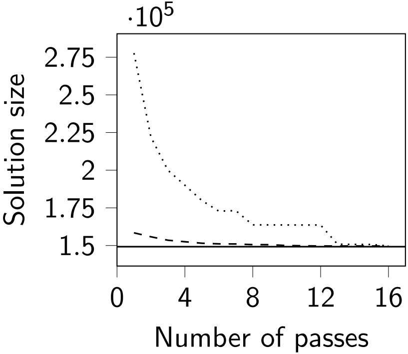
(b) kosarak.dat

Multi-Pass Emek-Rosén in Practice

Results (2/3)



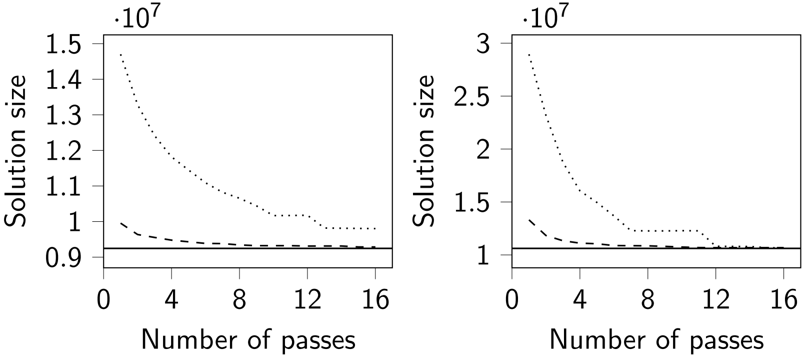
(a) webdocs.dat



(b) orkut-cmty.dat

Multi-Pass Emek-Rosén in Practice

Results (3/3)



(a) twitter.dat

(b) friendster.dat

Summary

- EMEK-ROSÉN works well in practice

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  - Slightly larger covers
  - Much smaller memory footprint
  - Faster

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  - Slightly larger covers
  - Much smaller memory footprint
  - Faster
- Solution quality can be improved with multiple passes

References I

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